On the minimal resolution conjecture for points in general position in the projective space.

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Abstract

Let $S = \{P_1, \ldots, P_s\} \subset \mathbb{P}^n$ be a set of points in general position and denote by $R = [x_0, x_1, \ldots, x_n]$ the homogeneous coordinate ring of $\mathbb{P}^n$. Then the homogeneous ideal I_S of these points has a minimal graded free resolution of the form;

$$0 \longrightarrow F_{n-1} \longrightarrow \ldots \longrightarrow F_2 \longrightarrow F_1 \longrightarrow F_0 \longrightarrow I_S \longrightarrow 0,$$

where $F_p = R(-d-p)^{a_{p-1}} \bigoplus R(-d-p-1)^{b_p}$, d is the smallest integer satisfying $s \leq \mathrm{h}^0(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(d))$ and $\binom{d+n-1}{n} \leq s < \binom{d+4}{n}$, with $a_p b_p = 0$ for $p = 0, 1, 2 \ldots, n-2$. Proving the existence of a resolution of the above form is the same as showing that the Betti numbers a_i and b_i satisfy $a_i b_i = 0$. This is achieved by showing that the map below is of maximal rank using the method of Horace.

$$\mathrm{H}^0\left(\mathbb{P}^n, \Omega_{\mathbb{P}^n}^{p+1}(d+1+p)\right) \longrightarrow \bigoplus_{i=1}^{s} \Omega_{\mathbb{P}^n}^{p+1}(d+1+p)|_{P_i}.$$

Key words: Betti numbers, maximal rank, method of Horace, minimal free resolution.